Journal of Engineering Physics and Thermophysics, Vol. 74, No. 5, 2001

SOME MODELS OF STATIONARY THERMOELECTRIC REFRIGERATORS

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UDC 621.382.53

Refrigerators based on the transverse effect of Peltier or Ettingshausen that operate under adiabatic conditions and also a refrigerator based on the Thomson effect are suggested.

An anisotropic thermoelectric refrigerating element and an Ettingshausen refrigerating element are distinguished by the fact that one of their lateral faces is thermostatted [1, 2]. It is assumed that this face is in ideal thermal contact with a thermostat and at the same time it is electrically insulated from the thermostat. In the present case, these two requirements are in contradiction with each other. Therefore, it becomes necessary to create such a structure in which there would be no need for thermostatic control of the lateral face. For this purpose, in the present work the author suggests refrigerating elements whose operating effects are the transverse Peltier effect or the Ettingshausen effect, while the lateral faces are adiabatically insulated from the environment.

The possibility of creating a refrigerator based on the Thomson effect is also considered.

1. Adiabatic Anisotropic Refrigerating Element (ARE). As the ARE material, we select a thermoelectrically anisotropic homogeneous semiconductor with temperature–independent kinetic coefficients. Then a heat-conduction equation can be written in the form

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \gamma = 0 , \tag{1}$$

where the temperature is considered to be two-dimensional (Fig. 1). The boundary conditions are as follows:

$$T\,(0, y) = T\,(l, y) = T_0 , \tag{2}$$

$$\left.\frac{\partial T}{\partial y}\right|_{\substack{y=0\\y=h}} - \beta T\,\Big|_{\substack{y=0\\y=h}} = 0 . \tag{3}$$

Conditions (2) mean the thermostatic control of the end faces of the ARE at the temperature T_0, while conditions (3) mean the adiabatic insulation of the lower and upper faces.

The solution of Eq. (1) with conditions (2) and (3) can easily be found by the Fourier method. This solution has the form

$$T\,(x, y) = T_0 - \frac{1}{2}\,\gamma x\,(x - l) + \frac{\beta}{2}\sum_{n=1}^{\infty}\frac{T_0 D_n - \frac{1}{2}\,\gamma C_n}{\sinh \delta_n h} \times$$

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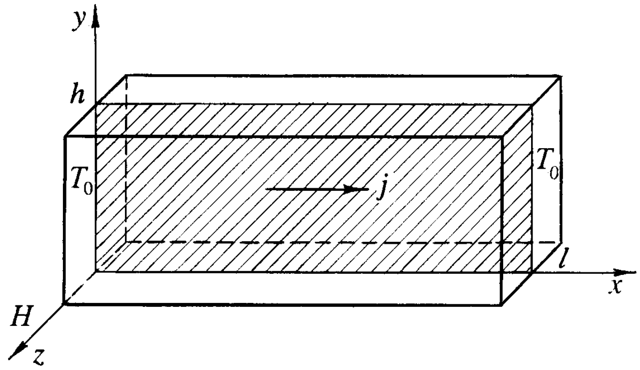

Fig. 1. Schematic diagram of the adiabatic ARE or the Ettingshausen adiabatic RE (in the case of the Ettingshausen RE: x is the trigonal axis, y is the binary axis, and z is the bisector axis). The faces $y = 0$ and $y = h$ are adiabatically insulated from the environment.

$$\times\left[\frac{\exp(-\delta_n h)-1}{\delta_n-\beta}\exp(\delta_n y)+\frac{\exp(\delta_n h)-1}{\delta_n+\beta}\exp(-\delta_n y)\right]\sin\delta_n x\,,$$

where $D_n = 2(1-(-1)^n)/(n\pi)$ and $C_n = -4l^2(1-(-1)^n)/(n\pi)$ are the coefficients of expansion of 1 and of the functions $x(x-l)$ into Fourier series in sines.

The temperature distribution over the face $y = 0$ is as follows:

$$T(x,0)=T_0-\frac{1}{2}\gamma x(x-l)+\beta\sum_{n=1}^{\infty}\frac{T_0D_n-\frac{1}{2}\gamma C_n}{(\delta_n^2-\beta^2)\sinh\delta_n h}[\beta\sinh\delta_n h+\delta_n(1-\cosh\delta_n h)]\sin\delta_n x\,.$$

Cooling will occur on condition that the third term in the last expression is negative.

We estimate the temperature at the point $x = l/2$ if a current flows in the negative x direction (Fig. 1). Let $\alpha_{12} = 10^{-4}$ V/K, $\chi = 10^{-2}$ W/(cm·K), $\rho = 10^{-3}$ Ω·cm, and $\delta_n h \geq 3$. We obtain

$$T(l/2,0)=T_0-\frac{1}{8}\gamma l^2-\frac{\beta l}{\pi}\sum_{n=1}^{\infty}\frac{T_0D_n-\frac{1}{2}\gamma C_n}{n}\sin\frac{n\pi}{2}\,. \tag{4}$$

The last expression is written on condition that $\sinh\delta_n h \approx \cosh\delta_n h$ and $\delta_n >> \beta$. Let $T_0 = 300$ K and $jl = 60$ A/cm; then, according to Eq. (8), $T(l/2, 0) = 270$ K, i.e., the temperature decrease is 30 K.

2. Ettingshausen Adiabatic Refrigerating Element (RE). The Ettingshausen RE on specimens with a thermostatted face was investigated, for example, in [3, 8]. As the RE material, Bi and BiSb-based alloys were used. The current was directed over the trigonal axis, the magnetic field was directed over the bisector axis, and the temperature drop was guided over the binary axis. The Ettingshausen cooling effect is of interest since it can be applied to the region of cryogenic temperatures. The author of the present work made a certain contribution to the investigation of the Ettingshausen effect on specimens with a thermostatted lateral face [9]. We have noted above that it is difficult to attain a reliable thermal contact between the lateral face and the thermostat.

Let the RE of the above-indicated orientation be made of a Bi single crystal. Assuming the kinetic coefficients of the RE material to be independent of coordinates and temperature, and heat conduction to be isotropic, we direct the current over the trigonal axis and the magnetic field, over the bisector axis (Fig. 1). Then the heat-conduction equation will be written in the form

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + 2\eta \frac{\partial T}{\partial y} + \gamma = 0 . \tag{5}$$

The boundary conditions are

$$T(0, y) = T(l, y) = T_0 , \tag{6}$$

$$\left. \frac{\partial T}{\partial y} \right|_{\substack{y=0 \\ y=h}} + \eta T \Big|_{\substack{y=0 \\ y=h}} = 0 . \tag{7}$$

The solution of Eq. (5) with conditions (6) and (7) will be as follows:

$$T(x, y) = T_0 - \frac{1}{2}\gamma x(x - l) + \frac{\eta}{2}\exp(-\eta y) \sum_{n=1}^{\infty} \frac{T_0 D_n - \frac{1}{2}\gamma C_n}{\varepsilon_n \sinh \varepsilon_n h} \times$$

$$\times [(\exp(-\varepsilon_n h) - \exp(\eta h))\exp(\varepsilon_n y) + (\exp(\varepsilon_n h) - \exp(-\eta h))\exp(-\varepsilon_n y)] \sin \delta_n x ,$$

where D_n and C_n are defined in item 1. We estimate the temperature at the point $(l/2, 0)$ from the formula

$$T(l/2, 0) = T_0 + \frac{1}{8}\gamma l^2 + \eta \sum_{n=1}^{\infty} \frac{\cosh \varepsilon_n h - \exp \eta l}{\varepsilon_n \sinh \varepsilon_n h} \left(T_0 D_n - \frac{1}{2}\gamma C_n \right) \sin \frac{n\pi}{2} .$$

Provided that $\eta h << \delta_n h$ and $h \approx l \approx 1$ cm, the last expression will be of the form

$$T(l/2, 0) = T_0 + \frac{1}{8}\gamma l^2 + \eta \sum_{n=1}^{\infty} \frac{T_0 D_n - \frac{1}{2}\gamma C_n}{\delta_n} \sin \frac{n\pi}{2} .$$

For numerical estimation, we select the following parameters: $\rho = 10^{-3}\ \Omega\cdot$cm, $QH = 2\cdot 10^{-4}$ V/K, $\chi = 10^{-3}$ W/(cm·K), $j = -16$ A/cm^2, and $T_0 = 80$ K. We obtain $T(l/2, 0) = 72$ K. Further work on the adiabatic RE of Ettingshausen must involve optimization of the parameters (material, dimensions, current, etc.).

3. RE Model Based on the Thomson Effect. The Thomson effect (TE) involves the liberation (absorption) of heat in the volume of a nonisothermal conducting medium through which an electric current flows. The heat is liberated (absorbed) per unit volume [10]:

$$q_{\mathrm{T}} = T \frac{d\alpha}{dT} \frac{dT}{dx} j .$$

The Thomson effect has long been discovered [11] but has not found practical application.

Below we suggest a phenomenological model of a stationary refrigerator based on the Thomson effect; in this model a temperature gradient and an electric current coincide in direction. Having set that the specific resistance and thermal conductivity of the RE material as well as the derivative $d\alpha/dT$ are constant and that the temperature distribution is one-dimensional, we write the heat-conduction equation in the stationary case in the form

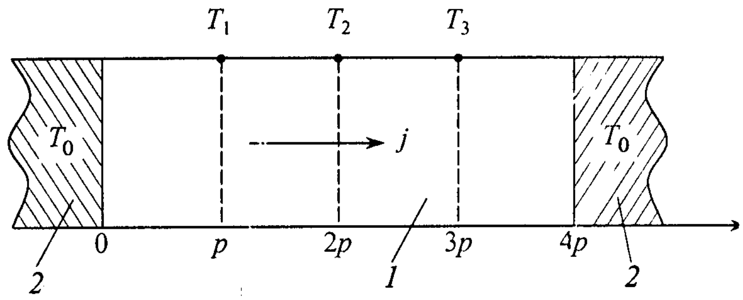

Fig. 2. Schematic diagram of the Thomson thermoelectric refrigerator: 1) specimen; 2) current leads to the specimen.

$$\frac{d^2T}{dx^2} - \delta T \frac{dT}{dx} + \gamma = 0 \tag{8}$$

and consider it simultaneously with the boundary conditions

$$T(0) = T(l) = T_0 \,. \tag{9}$$

To solve this problem, a grid method is used [12], according to which derivatives are replaced by their approximate values that are expressed in terms of the differences of the values of the function at separate discrete node points. As a result of these transformations, the differential equation is replaced by equivalent relations in finite differences.

Before proceeding to numerical estimations of the temperature at separate points, we note that the problem will be solved more exactly, the larger the number of subdivisions of the specimen length into equal segments, i.e., the smaller p. However, a large number of subdivisions necessitates the use of a computer, which results in an analyticity loss. Therefore, we restrict ourselves to a small number of subdivisions, assuming the RE length to be not very large.

Let us subdivide the RE length into four equal parts. Then Eqs. (8) and (9) lead to the system of equations for T_1, T_2, and T_3 (Fig. 2)

$$T_0 + T_2 - 2T_1 - \frac{\delta p}{2} T_1 (T_2 - T_0) + \gamma p^2 = 0 \,,$$

$$T_1 + T_3 - 2T_2 - \frac{\delta p}{2} T_2 (T_3 - T_1) + \gamma p^2 = 0 \,,$$

$$T_2 + T_0 - 2T_3 - \frac{\delta p}{2} T_3 (T_0 - T_2) + \gamma p^2 = 0 \,,$$

from which we find

$$T_1 = \frac{2T_0 + 3\gamma p^2}{1 - \delta p T_0/2} - \frac{1 + \delta p T_0/2}{1 - \delta p T_0/2} T_3 \,, \quad T_2 = \frac{(2 + \delta p T_0/2)\, T_3 - \gamma p^2 - T_0}{1 + \delta p T_3/2} \,, \quad T_3 = \frac{\sqrt{B^2 - 4AC} - B}{2A} \,,$$

$$A = 2\delta p \frac{1 + \delta p T_0/2}{1 - \delta p T_0/2} \,, \; B = (2 + \delta p T_0/2) \left(1 - \frac{\delta p}{2} \frac{2T_0 + 3\gamma p^2}{1 - \delta p T_0/2} \right) + (2 - \delta p T_0/2) \frac{1 + \delta p T_0/2}{1 - \delta p T_0/2} -$$

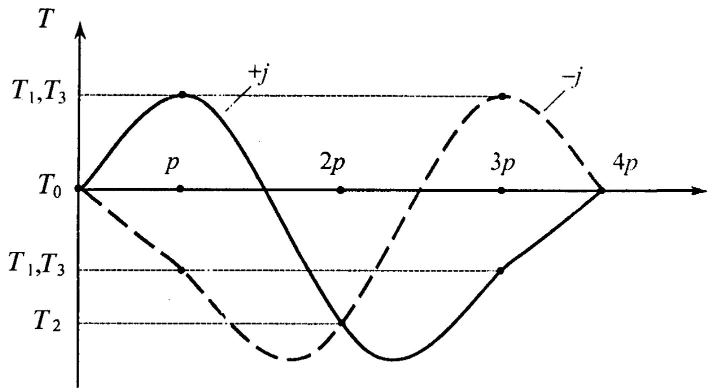

Fig. 3. Temperature distribution in the specimen of the Thomson refrigerator for the forward (+j) and backward (–j) directions of the electric current.

$$-\frac{\delta p}{2}\frac{1+\delta pT_0/2}{1-\delta pT_0/2}(T_0+\gamma p^2)+\left(T_0-2\frac{2T_0+3\gamma p^2}{1-\delta pT_0/2}\right)\frac{\delta p}{2}+\left(\frac{\delta pT_0}{2}\frac{2T_0+3\gamma p^2}{1-\delta pT_0/2}+\gamma p^2\right),$$

$$C=\frac{2T_0+3\gamma p^2}{1-\delta pT_0/2}\left((2T_0+\gamma p^2)\frac{\delta p}{2}-2\right).$$

Now we estimate numerically the temperatures T_1, T_2, and T_3. We set l = 1 cm, T_0 = 300 K, $\rho = 10^{-3}$ Ω·cm, $\chi = 10^{-2}$ W/(cm·K), $d\alpha/dT = 10^{-4}$ V/K^2, and j = 0.15 A/cm^2. For these parameters we will have γ = $2.3 \cdot 10^{-3}$ K/cm^2, $\delta = 1.5 \cdot 10^{-3}$ (K·cm)$^{-1}$, and $\delta pT_0/2 = 0.056$. Since $\delta pT_0/2 \ll 1$, we obtain approximately that $A = 2\delta p$, $B = 4$, and $C = -4T_0$ and, correspondingly $T_3 = T_0(1-\delta pT_0/4)$, $T_1 = T_0(1+\delta pT_0/4)$, and $T_2 = T_0(1-\delta pT_0/2)$. The temperature T_2 = 280 K. Thus, in the case considered we have a decrease of 20 K in the temperature.

We note the following circumstance. If in the expressions for T_1, T_2, and T_3 we replace j by $(-j)$, then T_1, T_2, and T_3 become different, which is apparently in contradiction with a physical situation. For example, it is obvious that T_2 must not depend on the direction of the current. This contradiction can be eliminated if we take into consideration that on replacing j by $(-j)$ the sign of the derivative dT/dx is also changed, so that the product jdT/dx in Eq. (8) does not change sign. At the same time, T_1 and T_3 change places, whereas T_2 remains the same. The dependence $T(x)$ is shown qualitatively in Fig. 3, from which it is clear that the point at which the temperature is minimum can be shifted relative to $x = 2p$.

The above calculations are of an illustrative nature and are called upon to show that the Thomson effect can serve for the purpose of cooling. Further work must be aimed at searching for optimum materials, currents, and specimen dimensions, which is not the subject of the present investigation.

NOTATION

T, temperature; x, y, z, axes of a Cartesian coordinate system; h, height; l, RE length; $\beta = \alpha_{12}j/\chi$; α_{12}, coefficient of transverse thermal e.m.f; $\varepsilon_n = (\delta_n^2+\eta^2)^{1/2}$; $\delta_n = n\pi/l$; n = 1, 2, 3, ..., summation index; $\eta = QHj/\chi$; Q, Nernst coefficient; H, magnetic field strength; $\delta = (d\alpha/dT)j/\chi$; $\gamma = \rho j^2/\chi$; α, thermal e.m.f depending linearly on temperature; ρ and χ, specific resistance and thermal conductivity; j, current density; q_T, Thomson volume heat density; $p = l/4$; T_1, T_2, and T_3, temperatures of the points in the Thomson RE; T_0, temperature of the thermostat.

REFERENCES

1. É. V. Osipov, *Solid-Body Cryogenics* [in Russian], Kiev (1977).
2. B. J. O'Brien and C. S. Wallace, *J. Appl. Phys.*, **29**, 1010–1012 (1958).
3. C. F. Kooi, R. B. Horst, W. F. Cuff, and S. R. Hawkins, *J. Appl. Phys.*, **34**, 1735–1742 (1963).
4. *Electronics*, **34**, 84 (1963).
5. H. J. Goldsmid, *G.-E.C. J.*, **31**, 110–113 (1964).
6. G. L. Gurthie, *J. Appl. Phys.*, **36**, 3118–3122 (1965).
7. T. C. Harman and T. M. Honig, *Thermoelectric and Thermomagnetic Effects and Applications*, New York (1967).
8. W. M. Yim and A. Amith, *Solid-State Electronics*, **15**, 1141–1165 (1972).
9. V. G. Okhrem, *Inzh.-Fiz. Zh.*, **40**, No. 1, 75–81 (1981).
10. A. F. Ioffe, in: *Semiconductor Elements* [in Russian], Moscow–Leningrad (1960), pp. 7–14.
11. L. I. Anatychuk, in: *Thermoelements and Thermoelectric Devices* [in Russian], Kiev (1979), pp. 28–30.
12. D. Yu. Panov, *Handbook on Numerical Solution of Partial Differential Equations* [in Russian], Moscow (1951).